

The Economics of Wild Goose Chases

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Abstract

Authority is a dominant feature of firms, often because supervisors have better information on appropriate task assignments. This paper addresses the provision of incentives in this context, where formal pay for performance can be used to align interests. As in Aghion and Tirole, 1997, agents fear that authority will be used to harm them. It is shown that when effort is feasible, authority is complementary with incentives when incentive provision is inexpensive, but harms incentive provision when incentives are difficult to provide. Existing empirical evidence suggests that authority enhances worker incentives rather than crowding out motivation as is often assumed.

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Most people follow orders in work. This is often because others - usually their bosses - are better informed about opportunities and tradeoffs than they are, and hierarchical authority is a defining feature of organizational life. Yet firms also need to motivate workers. Aghion and Tirole, 1997, argue that formal authority by superiors likely harms worker motivation, as workers have a greater fear of being given undesired tasks. I address the exercise of authority in a setting where pay for performance is feasible, and find that its impact on motivation depends on the ease of incentive provision. Specifically, greater authority enhances incentive provision when incentives are inexpensive to provide, but makes it more difficult when incentives cannot be easily attained. I also use the model to argue that existing empirical evidence is consistent with increased authority enhancing worker motivation rather than crowding it out.

The reason for authority here is that a principal knows more about productive opportunities that does the agent.¹ A good motivating example is where bosses assign clients (or markets) to their subordinates, but where the boss knows something about those clients. For example, fund raisers are assigned candidates for raising money by development directors. The fund raiser can exert effort on this, but the return to doing so is increasing in the quality of the target. Authority then has the conceivable advantage of assigning the highest quality target to the fund raiser.

Aghion and Tirole study the effect of authority on incentives where the conflict of interest over tasks arises from the agent receiving private benefits from certain activities. Here conflict of interest arises not because of such private benefits but because tasks vary in how well their returns can be measured - some are easily measured (*contractible tasks*) while some cannot be measured (*non-contractible tasks*).² (So in the fund raising example, some donors may be likely to give money over the short run, for which the fund raiser would be compensated, whereas others are long run targets and unlikely to give immediately.) I begin with a baseline model, where a principal privately observes the marginal return to effort for n contractible tasks, and for n non-contractible tasks. (For example, these are the potential clients for the fund raiser.) In the baseline model, all tasks have marginal return to effort drawn from a

¹Another literature on hierarchical authority deriving from Rosen, 1982, and more recently exemplified by Garicano, 2000, argues for the returns to placing the most able in senior positions in order to leverage their skills. Superiors have skills that their subordinates do not, they see more information, and ultimately have the potential to make better decisions.

²There is a short section on the role of pay for performance in Aghion and Tirole. See also Bester and Krahmer, 2008, for a case where pay for performance is allowed in the Aghion and Tirole setting. The insights of both are very different to the results here. Bolton and Dewatripont, 2011, provide a comprehensive survey of the literature on authority and incentives.

similar (uniform) distribution. As in Aghion and Tirole, effort is exerted before tasks are assigned.³ The metric for the importance of authority is n . In the first best, the agent is always assigned to the highest productivity task, and more authority (higher n) increases the expected marginal return to effort and so is complementary with incentive provision. Now suppose that the principal offers an incentive contract to the agent to induce her to exert effort, where she gets a share of verifiable (or “observed”) output. As the agent is only paid for tasks that can be measured, the concern arises that the agent may be assigned to a “wild goose chase”, a less productive non-contractible task, rather than the most productive task.⁴

The key issue for incentives, as in Aghion and Tirole, is *congruence*: how likely is it that the principal’s preferred choice is also preferred by the agent? Congruence here is endogenous and varies with both pay for performance and the principal’s span of authority. Specifically, increased span of authority is double edged for providing incentives - it allows more efficient assignments but also makes a “wild goose chase” more likely as congruence simultaneously falls. The effect of increasing the span of authority depends on an Incentive Multiplier, where

- When incentives are inexpensive to provide (in a welfare sense), increases in the principal’s span of authority make incentives even cheaper to provide, but
- When incentives are expensive to provide, increases in authority make incentives yet more expensive.

I further show that the exercise of authority can cause incentives to fail, where no contract can induce effort exertion. More strikingly, in the baseline model incentives always fail for any positive effort cost if the principal has a sufficiently large set of assignment choices even though all tasks are ex ante identical. The reason is that with a wide enough span of authority, there is (almost) never task congruence between the principal and agent. Notably, this incentive failure arises when the marginal product of effort is sufficiently *high*.

Perhaps the most consistent empirical finding in the agency literature is that pay for performance falls when supervisors hold authority. When effort can be induced in the model,

³As described below, the idea here is that much of what constitutes “effort” is durable, and more akin to human capital. However, in Section 5.5 I show that similar insights arise when the agent exerts effort after tasks are assigned but does not know the contractibility of those tasks.

⁴The incentive for a principal to renege to save on compensation costs is similar to the implicit contracting literature of Bull, 1987, Baker, Gibbons, and Murphy, 1992, and Powell, 2011, or its one-shot analog such as Waldman, 1984, and Kahn and Huberman, 1990. Also see Fehr et al, 2010, for experimental evidence on how supervisors abuse authority for their own ends.

a sufficient condition for authority facilitating incentives is that it reduces pay for performance, as arises in the empirical literature. On the basis of this, I argue that the ability of principals to make better task assignments may empirically outweigh any demotivating effects of authority on incentives.

Much of the recent agency literature has addressed instruments other than pay that could aid effort exertion. Foremost among these is intrinsic motivation, where agents inherently value their outputs. An issue in this literature is how pay for performance affects intrinsic motivation (Ariely et al, 2009, Deci et al, 1999). This paper offers a novel reason why pay for performance adversely affects intrinsic motivation - the agent begins to distrust the instructions of the principal when incentive pay is used, and (correctly) infers that she is being assigned to inefficient activities. Authority is more trustworthy without incentive pay. Furthermore, I show that when intrinsic motivation is present, the total effect of pay for performance on incentives depends on how well output is monitored - when monitoring is easy, total effort rises with incentive pay, while it declines when monitoring is hard.

The remainder of the paper is concerned with the robustness of these insights. Many of the insights generalize in a straightforward way, but there are two notable caveats. First, the results above arise in a setting where a single worker is assigned to one of n tasks, and n varies. In terms of understanding the implication of increasing the range of supervisor authority, a realistic extension would be to allow the number of agents to increase with number of available tasks. This allows us to distinguish between simply increasing the “scale” of the principal’s authority and the “scope” of his authority (where the number of options per worker rises). I show that what matters for more authority causing problems for incentives is not the scale of the manager’s discretion but rather its scope.

The other caveat involves distributional assumptions. The effect of increased authority in the baseline model is ambiguous because of two conflicting effects: (i) a better informed superior can make better choices, but (ii) workers increasingly distrust those choices. In Section 5 I show that this second effect need not arise with other distributional assumptions.

I begin by describing the model in Section 1. I then consider the symmetric baseline case in Section 2, showing how the abuse of authority constrains incentives, and also how the effect of greater span of authority on incentives depends on the ease of providing incentives. I also interpret the empirical evidence in the light of these results. Section 3 shows how intrinsic motivation is affected by pay for performance. Section 4 addresses institutional solutions to the potential abuse of authority. Section 5 considers the robustness of the results, and addresses continuous effort choices, optimal contracts, different interpretations of the span

of authority, task specific efforts, and other distributional assumptions. I conclude in Section 6.

1 The Model

A profit maximizing principal hires an agent to work on a single task. Productivity on that task depends on unobservable effort e , where e takes on a value of 1 or 0, where the marginal cost of effort of 1 is γ . Output is only produced in the event that effort of 1 is exerted.

There is a range of tasks to which the agent can be assigned. There are $2n$ such activities. The activities are of two types:

1. n of these activities produce an output that is contractible, and the return to effort on task i is given by d_i . The true value of d_i is privately observed by the principal, while the agent knows only that it is drawn from a Uniform distribution with support $[0, D]$.
2. The other n activities produce an output that is not contractible, and the return to effort on task i is given by b_i . The true value of b_i is privately observed by the principal, while the agent knows only that it is drawn from a Uniform distribution with support $[0, B]$.

As a concrete example, one can think of each draw as a potential client for the agent, or a potential market to enter.⁵ All draws are assumed to be independent. Assume that the principal assigns the agent to a task - formally assume that only the principal can distinguish between tasks.

There are three natural cases that fit the notion of non-contractibility:

- *Non-Monetary Benefits*: Many returns that firms receive from workers are not easily measured in monetary terms.⁶ Any task that generates non-monetary benefits would satisfy the assumptions below.

⁵There is no need for a stark dichotomy between those activities where the agent will be rewarded from those where she will not. Instead, the qualitative results carry over when there is variation in the probability that the agent will be rewarded for exerting effort. For example, tasks could be contractible with either probability \bar{p} or probability $\underline{p} < \bar{p}$. The \bar{p} (\underline{p}) tasks are labeled contractible (non-contractible) but all that matters is that there is variation in how likely it is that the agent will be rewarded for exerting effort.

⁶These benefits could be the kind of thing that affects the image or brand capital of the firm. Alternatively, they could be private benefits to the principal, such as a “glamor project” or activities that make the principal look good to the labor market.

- *Timing*: In many settings, effort exerted today takes time to pay off, such as asking a manager to explore a possible new market. If those payoffs are far in the future, the agent may be long gone from the organization when the returns arise, and the agent receives no benefits.
- *Risk*: Risk preferences may also generate conflict if some activities are unlikely to succeed, but have high payoffs conditional on success. These may be activities that the principal would like to explore, but a risk averse agent would prefer to avoid in favor of something more certain.

Contracts The agent’s pay can be conditioned on observed output.⁷ The principal offers the agent a pay for performance contract where the agent is offered a share $\beta \geq 0$ of observed output and a fixed fee (or salary) β_0 .

Monitoring One of the central themes of the agency literature is that output is not perfectly measured. I assume that the contractible task is measured with possible error. Consider the case where the agent is assigned to a task with return d_i . If she exerts effort of 1, I assume that output d_i is always observed, but if effort of 0 is exerted, output of d_i is observed with probability σ .

The worker is assumed to maximize expected waged minus effort costs. I assume effort is exerted prior to task assignment, where effort is not task-specific. The logic behind this is that much of what we might term “effort” is durable, and more like human capital that retains value over time. For example, a worker can acquire human capital about how the business operates, can prepare presentations, can reorganize departments etc, that have value across time and tasks. (For example, much human capital in a sales setting is not client-specific.) In this setting, she could collect the skills while on some task, but could be subsequently reassigned to a non-contractible task, and not garner the returns to her actions.⁸

Timing First, the principal offers the agent a contract with sharing rule β and fixed fee β_0 . If the agent rejects, the game ends. If she accepts, the agent then exerts effort or not.⁹

⁷Meaning output that can be observed by a third party.

⁸The case of task specific effort is addressed in Section 5.5.

⁹That the effort is taken in the absence of a task assignment is an abstraction - it could be that the agent is always initially assigned to some base task, but future reassignments affect how likely she is to be

Following this, the principal observes the realization of the $2n$ random variables, and assigns the agent to one of the activities. The principal does so to maximize profits at that point. Output is then realized, and the agent is paid according to observed output. At that point, the game ends.

I characterize the surplus maximizing Bayesian Nash equilibrium¹⁰ of the game. Because there are no restrictions on β_0 , and contracts are chosen before private information is realized, the principal maximizes ex ante surplus. Specifically, he chooses β to maximize surplus produced before the realizations of the b and d distributions are known, subject to his own incentives to allocate tasks after observing the productivities.

2 The Symmetric Case: $B = D = 1$

Begin by considering the symmetric case where the contractible and non-contractible tasks are drawn from the same distribution, whose support is normalized to 1.

The First Best The principal receives $2n$ draws from the unit uniform, and chooses the task with highest realization. The first order statistic has expected value $\frac{2n}{2n+1}$. Hence the expected surplus in the first best is

$$e\left[\frac{2n}{2n+1} - \gamma\right], \quad (1)$$

so that if $\gamma \geq \frac{2n}{2n+1}$, the agent should exert effort. Let $\gamma^* = \frac{2n}{2n+1}$. Trivially, γ^* is increasing in n . In this sense, the importance of authority and incentive provision seem complementary.

Agent Incentives The agent is only rewarded if assigned to a contractible task, in which case the expected benefit to exerting effort is $\beta(1 - \sigma)$ times its marginal product. She does not know whether the task is contractible, nor its marginal return, when choosing effort and so she will exert effort if and only if

$$\beta(1 - \sigma)prob(task = d)E\{d_i|task = d\} \geq \gamma, \quad (2)$$

rewarded for doing so.

¹⁰The effort choice of the agent depends on what she expects the principal to do. There is always one trivial equilibrium here, where the agent assumes that the principal will assign her to a non-contractible task, and in response exerts no effort. Given no effort exerted, the principal is indifferent over a task to assign the agent, and indeed it is a Bayesian Nash equilibrium to assign her to such a task. I ignore this issue here by considering the surplus maximizing Bayesian Nash equilibrium of the game. An alternative solution to this issue would be to assume that there is some small probability of the b and d outputs being attained without effort exertion.

where the agent's beliefs are determined by the surplus maximizing equilibrium.

Lemma 1 *The agent's incentive compatibility constraint is given by*

$$\beta(1 - \beta)^n \frac{n}{2n + 1} \geq \frac{\gamma}{1 - \sigma}. \quad (3)$$

The results of the paper largely derive from this condition. There are two conflicting pieces to it. First, the expected productivity of the contractible task, if assigned to such a task, is $\frac{2n}{2n+1}$, which is increasing in n . Second, the probability of being assigned to a contractible task is $\frac{(1-\beta)^n}{2}$, which is decreasing in both n and β . The product of the two effects yields (3) and the effect of authority on incentives is a horserace between these two effects.

The $\frac{(1-\beta)^n}{2}$ term is the analog to *congruence* in Aghion and Tirole - how likely is it that the principal's preferred choice is also preferred by the agent? Here congruence is endogenous for the reason that benefits to the agent (i.e., wages) cost the principal, whereas worker benefits are incidental at the task assignment stage in Aghion and Tirole. This endogeneity of congruence has the feature that both increased incentive pay or span of authority make congruence less likely.

2.1 The Incentive Multiplier

Consider the case when the agent can be induced to exert effort: some β exists to satisfy (3). It is useful to begin by considering surplus with effort exerted. This is computed in the Appendix to be

$$S(\beta) = \frac{n}{2n + 1}(1 - \beta)^{n+1} + \frac{n}{2n + 1}(1 - \beta)^n + \frac{n}{n + 1}[1 - (1 - \beta)^{n+1}] - \gamma. \quad (4)$$

Conditional on effort being exerted, this is decreasing in β as the agent is misallocated more. As a result, the principal will choose the lowest value of $\beta = \beta^{**}$ where

$$\beta^{**}(1 - \beta^{**})^n \frac{n}{2n + 1} = \frac{\gamma}{1 - \sigma} \quad (5)$$

Furthermore, if β^{**} declines in n , incentives can be provided more cheaply with a larger span of authority. I call the relationship between β^{**} and n the Incentive Multiplier, which is positive if $\frac{d\beta^{**}}{dn} < 0$. Proposition 1 immediately follows.

Proposition 1 *Assume that some β exists to satisfy (3). Then increased authority (higher n) reduces the cost of incentive provision if and only if*

$$\log(1 - \beta^{**}) + \frac{1}{n(2n + 1)} > 0. \quad (6)$$

This has an indeterminate sign because there are two conflicting effects of increasing the principal's options. First, the productivity of the best contractible task increases - this is the $\frac{1}{n(2n+1)}$ term. Second, the agent is more likely to be "cheated" by being given a non-contractible task. This effect is proportional to $\log(1-\beta^{**})$, and the total effect is determined by the sum of these two factors.

The Incentive Multiplier depends on β^{**} and n . For low enough β^{**} the Incentive Multiplier is always positive. Said another way, when incentives can be provided inexpensively (in a surplus sense) a greater span of authority further reduces the cost. Incentives are inexpensive to provide when γ and σ are low, so a greater span of authority is complementary with incentive provision when monitoring is easy and costs low. By contrast, when β or n are high, the Incentive Multiplier always reduces surplus.¹¹

In effect, exercising a greater span of authority imposes an externality on incentive provision. When incentives are inexpensive to provide, this externality is positive. By contrast, when incentives are hard to provide, a greater span of authority imposes a negative externality on incentive provision.

2.2 The Failure of Incentives

Incentives can only be provided if there exists some β such that (3) holds. Does such a value of β^{**} exist? The maximized value of $\beta^{**}(1-\beta^{**})^n$ has a value of $\beta^{**} = \frac{1}{1+n}$. Lemma 2 immediately follows.

Lemma 2 *Incentives can be provided if and only if*

$$\frac{1}{2n+1} \left(\frac{n}{n+1}\right)^{n+1} \geq \frac{\gamma}{1-\sigma} \quad (7)$$

If this condition does not hold, there exists no incentive contract that can induce effort exertion. It is clear that this condition can fail if the costs of effort are high, or monitoring is poor.

For example, consider the case where the principal chooses between one contractible and one non-contractible task. When $n = 1$, the first best requires effort exertion for $\frac{\gamma}{1-\sigma} \leq \frac{2}{3}$.

¹¹Note however that β cannot be too high, or else the feasibility constraint in (7) is violated, so is it the case that for feasible levels of β , the required β can be increasing in n ? The answer is yes for any n . For $\beta = \frac{1}{1+n}$, $\log(1-\beta) + \frac{1}{n(2n+1)} < 0$ for any finite n so there is always a range of feasible levels of β where the incentive multiplier is negative.

However, from (7), incentives fails if $\frac{\gamma}{1-\sigma} > \frac{1}{12}$.¹² Furthermore, “enough” authority always eliminates incentives in Proposition 2.

Proposition 2 *Let $\gamma^{**}(n)$ solve (7) with equality. Then $\frac{d\gamma^{**}}{dn} < 0$. Furthermore, $\gamma^{**} \rightarrow 0$ as $n \rightarrow \infty$ so that incentives always fail if n is sufficiently large for any $\gamma > 0$ and $\sigma \geq 0$.*

In words, more options reduces the feasibility of effort ($\frac{d\gamma^{**}}{dn} < 0$) and a sufficiently large set of options for the principal always eliminates incentives.¹³ Yet this is precisely when the return to the agent’s effort is *highest*.

The reason that incentives always fail here is because the probability that the agent is assigned to a contractible task $\frac{(1-\beta)^n}{2}$ is decreasing in n and converges to 0 for any positive β .¹⁴ In the language of Aghion and Tirole, there cannot be *congruence* between the principal and agent with sufficient options and the agent never recovers her effort cost.¹⁵ Note that this result arises for any common distribution for the two kinds of tasks where there is a finite upper bound. For n gets large enough, there is never enough difference between the two first order statistics (the best contractible and the best non-contractible) to result in congruence.

The outcome of this section is illustrated in Figure 1, where optimal incentives (β^{**}) are plotted against n . The downward sloping hashed line gives the feasibility constraint, $\beta^{**} = \frac{1}{1+n}$, and only outcomes that lie below this line are feasible. I then distinguish between two cases - where incentive costs rise in n at $n = 1$, and where they do not. First consider the case where $\log(1 - \beta^{**}) + \frac{1}{n(2n+1)} < 0$ at $n = 1$ so $\log(1 - \beta^{**}) < -\frac{1}{3}$ and the Incentive Multiplier is always negative. Second, consider the case where $\log(1 - \beta^{**}) \geq -\frac{1}{3}$: here more

¹²The returns to the two tasks are independent. Positive correlation in returns make incentive provision harder. To see this, consider the simplest case where $n = 1$ but with probability ϕ the returns of the two tasks are independent, and with probability $1 - \phi$ the returns are identical. Then the feasibility condition becomes $\frac{\phi}{12} \geq \frac{\gamma}{1-\sigma}$, which is harder to satisfy than (7) with $n = 1$.

¹³The baseline model assumes an equal number of each type of task. Consider a simple alternative where the ratio of contractible to non-contractible tasks is $k > 1$, so there are n non-contractible tasks and kn contractible tasks. Then the feasibility condition is given by $\frac{k}{(k+1)n+1} \binom{n}{n+1}^{n+1} \geq \frac{\gamma}{1-\sigma}$. Not surprisingly, this is easier to satisfy than in the balanced case. However, the left hand side continues to declines in n and converges to 0, so more options for the principal both make effort more desirable yet less feasible.

¹⁴When the principal has few options, the marginal productivity of the best contractible and non-contractible tasks can be quite different. When the principal has more options, the expected difference between the productivities of the best task of each type becomes small, and so she is unlikely to be assigned to the contractible task if incentive pay is used.

¹⁵This is also reminiscent of Cremer, 1995, where better information held by the principal can erode incentives.

options for the principal initially reduces required incentive payments, and greater span of authority is complementary with incentive provision. However, even in that case, at some point n rises by enough to make $\log(1 - \beta^{**}) + \frac{1}{n(2n+1)} < 0$ and so eventually the two variables become positively related - incentives are harmed by more authority - and eventually hit the feasibility condition, but at a higher n than for the poor monitoring case.

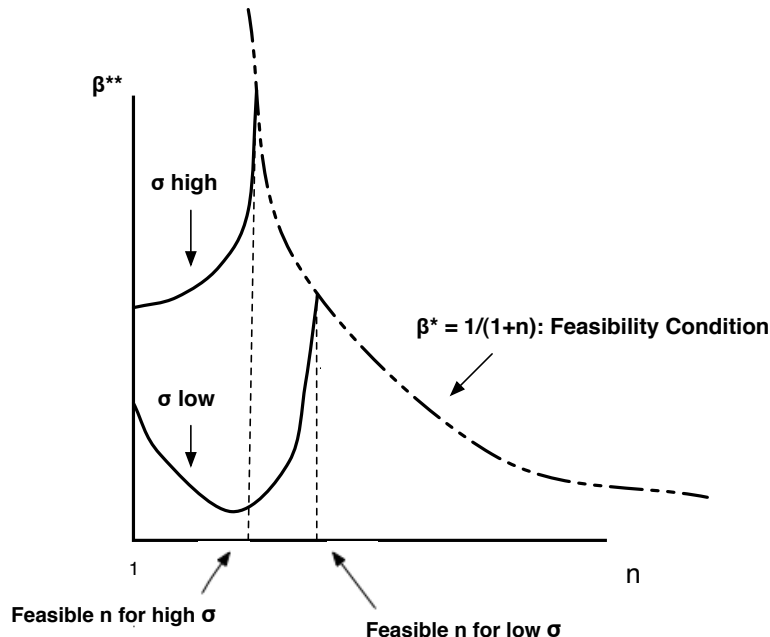


Figure 1: **The Incentive Multiplier**

Interpretation The exercise above concerns allowing the principal a wider span of assignment activities, which is being interpreted as greater authority. Yet the worker is not formally delegated control at all. A simple reinterpretation allows this. Specifically, assume that there is some larger set N of tasks that the agent could do, where n measures the number that the principal can assign and the remainder are under the control of the agent. Then assume that with probability $\frac{n}{N}$ the principal gets to assign the agent, and with probability $\frac{N-n}{N}$ the agent gets to choose, where she chooses randomly among those assigned to her, as she knows no better. With this interpretation, the total number of options does not change, but who chooses does, and may accord better with the empirical evidence below. The marginal impact of authority is qualitatively similar with this interpretation.

2.3 Empirical Observations

Perhaps the most consistent finding in the literature on pay for performance is that workers are less likely to have formal incentives when supervisors have authority over their actions. This is shown in Table 1 in a wide variety of settings.

Table 1: *The Relationship between Supervisor Authority and Worker Pay-For-Performance.*

Authors	Occupation/Sample	Result
McLeod and Parent (1999)	National Survey (U.S.)	< 0
Nagar (2002)	Bank Branch Managers (U.S.)	< 0
Colombo and Delmastro (2004)	Manufacturing Workers (Italy)	< 0
Foss and Lauren (2005)	Managers (Denmark)	< 0
Wulf (2007)	Division Managers (U.S.)	< 0
De Varo and Kurtulus (2010)	National Sample (Britain)	< 0
Ghosh, Lafontaine, and Lo (2011)	Sales Force Workers (U.S.)	< 0

This paper argues for a causal link between authority and pay for performance, and offers two insights, where the interpretation depends on whether incentives can be provided or not.¹⁶

1. The empirical relationship above shows that authority reduces incentive *intensity* - it does not necessarily imply that authority reduces incentives. Consider the case where incentives can be provided above. The Incentive Multiplier implies that authority facilitates incentive provision only if it *reduces* incentive pay. As supervisors make choices that increase the marginal returns to effort, less pay for performance is necessary. As a result, these data may argue for the complementarity of authority and worker incentives, rather than authority crowding out worker willingness to exert effort, as is often assumed.
2. However, some care must be taken here. Consider the results on incentive failure. At some point, the exercise of authority cannot co-exist with any effective pay for performance, and so it could be that an absence of pay for performance arises because effort disappears. If so, this extends the insights of Aghion and Tirole into a setting where pay for performance could potentially align interests.

¹⁶There are other possible reasons for this correlation. One could simply be that when efforts can be observed, supervisors tell agents what to and monitor their inputs, whereas an inability to observe inputs may render authority ineffective and hence require the use of pay for performance.

2.4 Effect on Surplus

So far I have only considered the effect of authority on the ability to induce effort exertion. Consider the case where effort can be induced by the agent. Changing the number of task assignment options has implications for surplus in (4) beyond the Incentive Multiplier. First, the principal finds better tasks on average - the $\frac{n}{2n+1}$ and $\frac{n}{n+1}$ terms reflect the value of more information - which increases surplus. Second, as n increases the principal misallocates the agent more to the non-contractible task, which will reduce surplus. This arises through varying n for the $(1 - \beta^{**})^{n+1}$ and $(1 - \beta^{**})^n$ terms. Finally, there is the Incentive Multiplier.

It should not be surprising that the aggregate effect of more information on surplus is ambiguous. There is one positive effect (the principal gets better draws), one that is negative (task assignments are more distorted for fixed β), and one which is ambiguous, the Incentive Multiplier. In general, this cannot be signed. However, there is a little more that we can say here.

Proposition 3 *Assume that incentives can be provided to the agent. Surplus is increasing in n when incentives are inexpensive to provide (β close to 0) but decreasing in n when incentives are sufficiently expensive to provide (close to $\beta = \frac{1}{n+1}$).*

3 Intrinsic Motivation

There has been considerable interest recently in the issue of intrinsic motivation, and the effect that it has on optimal pay for performance (Delfgaauw and Gur, 2003, 2006, Benabou and Tirole, 2003, Besley and Ghatak, 2007, and Prendergast, 2007). A central question in this literature is whether using pay for performance demotivates workers through reducing their intrinsic motivation. This paper offers a simple reason why pay for performance can demotivate, namely, that it causes workers to trust their bosses less. If this effect is large enough relative to the usual motivating effects of pay for performance, workers have less incentive to exert effort.

To see this, consider a scenario where the worker intrinsically values output Y at vY , where $v < 1$. Note here that the agent has the same objective as the principal (if muted), and values both the contractible and non-contractible outcomes. Expected output is $\frac{n}{2n+1}(1 - \beta)^{n+1} + \frac{n}{2n+1}(1 - \beta)^n + \frac{n}{n+1}[1 - (1 - \beta)^{n+1}]$ and so the incentive compatibility constraint

becomes

$$v \left[\frac{n}{2n+1}(1-\beta)^{n+1} + \frac{n}{2n+1}(1-\beta)^n + \frac{n}{n+1}[1 - (1-\beta)^{n+1}] \right] + (1-\sigma)\beta(1-\beta)^n \frac{n}{2n+1} \geq \gamma. \quad (8)$$

The piece that is new here is that β affects the agent's perception of output. Expected output is declining in β (holding effort constant), and so intrinsic incentives are harmed. Specifically, if U is the utility of the agent, then holding effort constant,

$$\frac{dU}{d\beta} = e(1-\beta)^n \left[(1-\sigma) \left(1 - \frac{n\beta}{1-\beta} \right) - \frac{vn^2\beta}{(2n+1)(1-\beta)} \right]. \quad (9)$$

Furthermore, the total effect of pay for performance depends on the ability to monitor - specifically, when monitoring is poor, the incentive to exert effort always falls, while if monitoring is good, incentives rise. Specifically, this term is negative for large σ and positive for small σ . In words, the marginal return to exerting effort falls in β if monitoring is poor but increases if monitoring is good.

4 Potential Solutions

The principal's preferences have largely been taken as given here, where he has an incentive to abuse his authority to reduce the agent's pay. Yet there are conceivably ways to limit the principal's interest in doing so.

Fixed Wage Bills In settings where there is more than one agent, a possible solution is to use some form of relative performance evaluation with a fixed wage bill, through something like a tournament. In this way, the only discretion that the principal holds is over who gets which rewards, rather than the total allocation. To the extent that fixed wage bills do not result in collusion by agents, these can alleviate the problem.

Bureaucracy Bureaucracy generally refers to the use of rules over allowing discretion in firms (Milgrom, 1988). Consider a scenario where the principal can commit to only choose from a predetermined random set $2m$ of tasks, where $m \leq n$. This choice occurs at the same time as the contract choice in the timing above. It should be obvious how restricting authority can improve incentives, given the results of the last section.¹⁷ First consider the

¹⁷See Rantakari, forthcoming, for another case where limited control on authority can aid effort exertion, and Friebel and Raith, 2004, for other work on how rules mitigate the misuse of authority.

case where the feasibility constraint is violated. One way to allow effort exertion is to restrict tasks to some m no higher than the largest m where

$$\frac{1}{2m+1} \left(\frac{m}{m+1} \right)^{m+1} \geq \frac{\gamma}{1-\sigma}. \quad (10)$$

Now consider the case where incentives are feasible. Remember from above the effect of n on surplus is indeterminate. As a result, it is hard to make concrete statements about the extent of bureaucratic restrictions when effort is feasible. However, from Proposition 3 surplus is decreasing in n close to the maximum feasible level $\beta = \frac{1}{1+n}$. As a result, the principal will also optimally restrict his options in this case so that bureaucracy will arise in some settings even when effort is feasible.

Delegation An alternative to supervisor authority is to allow the agent to choose, where she randomly chooses a task. Call this delegation.¹⁸ Up to now, little has been said about whether the agent knows the contractibility of tasks. If she does not know, she has a 50% chance of choosing a contractible task and her return to effort is $[(1-\sigma)\frac{1}{4} - \gamma]$, while if she knows whether a task is contractible, she will choose a contractible task and receive marginal return of $[(1-\sigma)\frac{1}{2} - \gamma]$. To give delegation its best chance, consider the case where the agent knows if tasks are contractible and $\frac{1-\sigma}{2} > \gamma$, in which case the agent will exert effort if she chooses the task. Then delegation is optimal¹⁹ only if $n > n^*$ where $\frac{1}{2n^*+1} \left(\frac{n^*}{n^*+1} \right)^{n^*+1} = \frac{\gamma}{1-\sigma}$.²⁰

Payments to Third Parties In the baseline model, the principal gains when the agent is not paid. But the agent not being paid is easily contractible, so another solution may be to penalize the principal whenever that happens, where the principal makes a transfer to a third party when incentive pay is 0. (Third parties are necessary here in order to retain incentives

¹⁸Note that delegation is identical to the principal simply randomizing over tasks. This tie is easily broken by imagining that the agent has some private benefit from carrying out tasks that only she knows about, where that private benefit is small enough not to overturn the optimal allocation rule.

¹⁹There is a better outcome than pure delegation, namely probabilistic delegation. Let M^d be the marginal return to exerting effort when he has control over the task carried out, and let M^p be her marginal return when the principal allocates tasks. Then delegation is only relevant if $M^p \leq \gamma$, but $M^d > \gamma$. But as the principal makes better allocative decisions than random choice of the agent, there is no reason to keep the agent's incentive compatibility constraint slack. Instead, the optimal probability of delegation is given by ρ^* , where $\rho^* M^d + (1 - \rho^*) M^p = \gamma$.

²⁰There is another conceivable reason why the agent may be delegated control, namely, that holding effort constant, the marginal surplus from random choice exceeds that of the principal choosing. This cannot occur because the principal never induces marginal returns worse than random choice.

to agents.) Or said another way, the principal is only rewarded on profits excluding wage costs, as in Zbojnik, 1998. In this way, his incentive to cut wage payments can be reduced.

More Complex Mechanisms So far it has been assumed that only the principal can assign tasks, through the assumption that only he can distinguish between tasks. If this assumption is dropped, it may be possible to design mechanisms to improve efficiency. In the Appendix, I show that if a mechanism designer can identify tasks and more complex mechanisms are allowed, the first best can be approximated. This also requires deep pockets for the principal. In this mechanism, the principal reports the realizations of the d and b vectors to a mechanism designer, and both the implemented task and payments are contingent on the reports. By using a Becker-like mechanism - investigate all states with small probability to get truth-telling - the principal can be induced to tell the truth over the productivities of all contractible tasks. Given this information, the designer then taxes the principal for implementing a non-contractible task by exactly the wage savings he would have received by assigning the worker to the best contractible task. In this way, the first best can be approximated.²¹

5 Robustness

In this section, I consider the robustness of the results to other assumptions.

5.1 Continuous Effort Choices

The effort choice here is discrete. An alternative would be to allow continuous choice of effort $e \geq 0$ where output is produced with probability e at cost $C(e)$, where $C'(e) > 0, C''(e) > 0$,

²¹The plausibility of this mechanism is debatable. First, it requires that the mechanism designer choose the task, which implies identifying whether the appropriate “task” has been implemented. While the outcomes of tasks may be sometimes easily identifiable, the tasks themselves are often so amorphous and fluid that their ex ante identification may be simply too difficult for a third party. Second, the mechanism requires large transfers from the principal to the agent, requiring deep pockets for the principal. Finally, in reality the mechanism may be too complex for the agent to compute both its value to her and how it solves the principal’s problems. The spirit of the paper is one where agents are poorly informed about the technology used by the firm, and this may go well beyond knowing the realization of the $2n$ random variable. Such knowledge is required for the agent to value the mechanism. For these reasons, the use of this kind of mechanism is likely to be limited.

and $C'(0) = 0$. In this case, the choice of effort is given by

$$\beta^{**}(1 - \beta^{**})^n \frac{n}{2n + 1} = \frac{C'(e)}{1 - \sigma} \quad (11)$$

The insights above continue to hold here, but in a more continuous way. The left hand side of (11) determines the equilibrium level of effort, which reaches a maximum at $e^{max} = C'^{-1}(\frac{1-\sigma}{2n+1}(\frac{n}{n+1})^{n+1})$. Effort increases in β up to $\beta = \frac{1}{1+n}$ and falls beyond that. Furthermore, $e \rightarrow 0$ as $n \rightarrow \infty$ as above.

Deriving the optimal contract in this case has one additional complication. With discrete costs, the principal's objective is to choose the lowest β subject to $e = 1$. Here the principal has a continuous tradeoff when increasing β : it makes output more likely (as e rises), but also causes more misallocation, where the principal in general chooses an interior solution. Subject to this additional complication, the results above continue to hold.

5.2 Optimal Output-Based Contracts

So far, I have assumed linear contracts, where the share of output obtained by the agent is independent of output produced. A relevant question here is the extent to which the results depend on the assumption of linearity.²²

Proposition 4 *Consider a more general contract where the agent receives a transfer $\beta(y)$ when output of y is observed. The unique optimal contract is linear in y for any n .*

5.3 The Span of Authority

The model offers one view of the principal's span of authority - namely, he assigns a single worker to one task among n . The purpose of this section is address whether the idea that more authority can harm incentive provision depends on this stark assumption. The comparative static above involves two features which are relaxed here. First, some tasks are not carried out in equilibrium - the "other" $n - 1$. Second, increasing n both increases the total number of tasks possible (a "scale" effect), and simultaneously the number of tasks per worker (a "scope" effect). To address these issues, I consider two other cases involving a return to assignment.

²²Linearity in optimal contracts is unusual, except in the well known Holmstrom and Milgrom (1992) setting. It arises here because if F is the CDF and f the density function of the first order statistic for the unit uniform with n draws, $\frac{F(y-\beta^*(y))}{f(y-\beta^*(y))} = \frac{y-\beta^*(y)}{n}$ is linear in $\beta(y)$ and so the optimal contract is linear.

5.3.1 More Tasks and Agents

One concern with the exercise of increasing n above is that more potential tasks do not allow more agents to do them. Here I also allow the number of agents to simultaneously change.

It is difficult to attain closed form solutions for small sample order statistics other than the best and worst elements so here I compare two cases where a closed form is easily attained. First, I consider the case of $n = 1$ above where there are two tasks and one worker. I compare that to the case where there are a large number of tasks *and* workers, by considering the limiting case where $n \rightarrow \infty$ but now a fraction t of those tasks is carried out. In other words, the ratio of workers to tasks is constant at t . A useful benchmark given the case of $n = 1$ above is constant returns to scale: $t = \frac{1}{2}$, where half of all tasks are carried out. This is a useful benchmark as this is the case where the scale of the principal's authority (n) has been increased, without changing its scope (t). Proposition 5 illustrates the critical value of t below which increased authority harms incentives.

Proposition 5 *Let β_1 be the smallest value to solve $\frac{\beta_1(1-\beta_1)}{3} = \frac{\gamma}{1-\sigma}$ and assume incentives can be provided with $n = 1$. Define t^* by*

$$\frac{2(1 - \beta_1)}{3} = 1 - \left(\frac{2(1 - t^*)}{2 - \beta_1} \right)^2, \quad (12)$$

where $t^ < \frac{1}{2}$. For all $t \geq t^*$ incentives per worker are cheaper to provide than with $n = 1$ while if $t < t^*$, they are more expensive.*

In words, incentives become harder to provide only if the number of possible assignments per worker increases - with constant returns to scale ($t = \frac{1}{2}$), it always becomes easier to provide incentives. This result illustrates that what matters for the potentially harmful effect of authority on incentives is not scale, but rather scope.

5.3.2 Matching Ability to Tasks

So far, there has been some redundancy of tasks, in the sense that tasks remain unstaffed in equilibrium. However, this is not necessary for the problem to arise. Consider the case where there is one task of each type ($n = 1$) and where there are two workers.

With no other additions to the model, there is no relevant sense of authority as it does not matter who does which one. However, now assume that the agents vary in their "ability", meaning the marginal return to their effort. Specifically, assume that one agent has marginal return to effort $a > 1$ times that of the other agent, where the other's is normalized to 1

as in the baseline model. Then the first best involves assigning the more able agent (the a one) to the task with the highest marginal productivity. If the highest productivity task is non-contractible, then this will always occur. However, if the highest productivity task is contractible, then at the point of task assignment, the principal receives $a(1 - \beta)d - b$ by assigning workers efficiently, and $ab - (1 - \beta)d$ by inefficient assignment. Then the principal assigns efficiently only if $(1 - \beta)d - b > 0$, which is identical to the baseline model. Hence the logic of the distortion in the previous section carries over to this assignment problem.

5.4 Other Distributional Assumptions

More authority potentially demotivates here because congruence falls for a fixed β . In this setting, one further stark result arises - a principal with sufficient options always causes incentives to fail. Though the setting for this is far from pathological - identical distributions with a common upper bound - this result does not necessarily arise with other distributions. I show this here in two ways.

5.4.1 Different Uniform Distributions

First, I allow one of the uniform distributions to have higher expected returns than the other. There are two cases - (i) where the non-contractible activities are ex ante more productive, and (ii) where the contractible returns are more productive. I consider each in turn.

When Non-Contractible Tasks Have Higher Expected Return Assume that $B > 1$ while D remains equal to 1. Then the principal is more likely than in the benchmark above to assign the agent to a non-contractible task. As a result, incentive compatibility is harder to satisfy than before, as shown in the Appendix, and the problems that harm effort exertion in the baseline hold with greater force.

When Contractible Tasks Have Higher Expected Return Now consider the case where $B < 1$ and $D = 1$. This case is more conceptually distinct, as there is now “daylight” between the best contractible task’s productivity and that of the non-contractible task. The effect of more pay for performance on the principal’s incentive to distort remains unchanged. The key issue for whether the qualitative nature of the comparative statics on n change is not whether D exceeds B , but rather whether $D(1 - \beta)$ exceeds B . If $D(1 - \beta) < B$, the

model is qualitatively no different from before.²³ If, however, $D(1 - \beta) \geq B$, the incentive constraint is given by

$$\beta \left[\frac{n}{n+1} D \left[1 - \left(\frac{B}{D(1-\beta)} \right)^{n+1} \frac{n}{2n+1} \right] \right] \geq \frac{\gamma}{1-\sigma}. \quad (13)$$

Unlike the previous sections, the incentive constraint now becomes easier to satisfy holding β constant as n increases. However, this condition does not help as β is endogenous, so to make more progress I identify a lower bound on β .

Proposition 6 *The lowest possible value of β is given by $\underline{\beta} = \frac{\gamma}{D(1-\sigma)}$. A necessary condition for increases in n to relax the feasibility condition is*

$$D(1 - \underline{\beta}) \geq B. \quad (14)$$

Proposition 6 illustrates the possibility of greater scope of authority improving congruence. If (14) holds, then for sufficiently large n , it will be the case that $D(1 - \beta) > B$, and so more scope of authority increases the likelihood of being assigned to a contractible task, even though the principal has to give a piece of its returns to the agent.²⁴

5.4.2 Other Distributions

Consider a more general distribution, where the two kinds of tasks have identical unbounded distributions, but where the CDF of the distribution of the most productive task of each type is $F_n(z)$ defined from 0 to ∞ , with density $f_n(z)$. Then with a linear contract, the worker's incentive compatibility constraint is given by $\int_0^\infty \beta z F_n((1 - \beta)z) f_n(z) dz \geq \frac{\gamma}{1-\sigma}$. Increasing β continues to have the effect of making an assignment to a contractible task (weakly) less likely, so the first implication above continues to hold. However, it is not necessarily the case that increases in the scope of the principal's authority makes "wild goose chases" more likely.

²³The incentive constraint is $\left(\frac{D}{B}\right)^n \beta^{**} (1 - \beta^{**})^n \frac{n}{2n+1} \geq \frac{\gamma}{1-\sigma}$. Conceptually this is no different from above - incentives may be somewhat easier to achieve than in the symmetric case (as $\frac{D}{B} > 1$), but it remains the case that incentives fail with enough options for the principal.

²⁴Note however, that there are three caveats. First, if monitoring is poor (σ sufficiently large), it can never be the case that (14) holds. Second, even if monitoring is perfect, it is not enough that the upper bound of the contractible distribution exceeds the non-contractible one, instead it must exceed it by the cost of effort for (14) to hold. Finally, the condition above implies that when the principal's information set becomes sufficiently rich, further increases in n make incentives easier to provide. It does not imply that for lower n .

To see this, it is worth considering two distributions, the Freschet and Gumbel.²⁵ Proposition 7 shows that greater scope of authority does not lead to more misallocation in these cases.

Proposition 7 *The probability of being assigned to a contractible task, holding β fixed, is independent of n for both the Freschet and Gumbel distributions.*

Consequently, greater scope of authority is always beneficial to the provision of incentives, for the reason that the principal is getting better and better draws - so β can fall - with no reduced likelihood of being assigned to a “wild goose chase” for a given β .

5.5 When Efforts are Task Specific

Sometimes efforts are exerted after agents have been assigned to tasks. This section offers a modified version of the game that shows that the insights above continue to hold when efforts are task specific. The setting is one where (i) the principal assigns the agent to a task after observing productivity, (ii) the agent chooses effort after assignment, but (iii) the agent does not know the contractibility of her task. So for example she is unsure as to whether the activity returns monetary benefits now or in the future.

There is one additional complication that this changed timing gives rise to, specifically, the principal can potentially reveal the return to exerting effort (Maskin and Tirole, 1990). Cheap talk will not suffice as a means of persuading the agent - instead, credible information must involve a cost to the principal. This cost is in the form of a discretionary transfer or “gift” made to the agent before effort exertion. As effort is binary, and we are considering pure strategies, there is never more than one such credible transfer offered in equilibrium.²⁶ Accordingly, consider the case where after observing the realizations of the marginal productivities, the principal can, at his discretion, offer g to the agent.

²⁵The reason is that for order statistics that have a limiting non-degenerate distribution, the distribution of the first order statistic must converge in n to one of three distributions - Weibull, Freschet, or Gumbel. The commonly used Normal, Log-Normal, Exponential, Gamma, Log, and Weibull distributions converge to Gumbel, while the fatter tailed Cauchy, Pareto, and Freschet converge to Freschet. I focus on the latter two, as these retain their shape as n increases, in the sense that the first order statistic of a Gumbel is Gumbel, and similarly for the Freschet. As a result, by considering these two distributions I can make statements about both small n and the limiting case.

²⁶The identity of the task cannot be used to signal here as all tasks are identical to the agent.

Modified Timing: First, the principal offers the agent a contract with sharing rule β , a fixed fee β_0 , and a discretionary transfer g . If the agent rejects, the game ends. If she accepts, the principal privately observes the realization of the $2n$ random variables. He then assigns the agent to one of the activities and chooses whether to offer g . The agent then exerts effort or not. Output is then realized, and the agent is paid according to observed output. At that point, the game ends.

Consider an equilibrium of the form where receipt of g results in effort exertion, but a failure to receive g results in no effort. The principal offers g only if his profit from the agent exerting effort from its receipt is at least g . As there are two kinds of tasks, the principal offering g implies that either $\max\{b_i\}$ or $(1 - \beta)\max\{d_i\}$ exceeds g . By change of variables this is equivalent to the principal revealing to the agent that either the best contractible task has productivity above y^* or the best non-contractible task has productivity above $(1 - \beta)y^*$. The firm then chooses y^* to maximize ex ante surplus.

Lemma 3 *The agent's incentive compatibility constraint when the principal can choose y^* is given by*

$$R(y^*(\beta))\beta(1 - \beta)^n \frac{n}{2n + 1} \geq \gamma, \quad (15)$$

where $R(y^*(\beta)) = \frac{1 - y^{*(2n+1)}}{1 - (1 - \beta)^n y^{*2n}}$. Furthermore, $R(0) = 1$, $R(1) = 0$, and $\frac{dR}{dy^*} > (<)0$ at $y^* = 0(1)$.

This is similar to the initial incentive constraint - it adds only the term $R(y^*(\beta))$ to agent's incentives. Incentives can always be relaxed by offering a small gift, as R is increasing in y^* at $y^* = 0$. Hence gifts can be used to overcome some of the issues in the previous sections.²⁷ Note also that R is decreasing in β so this model has the same qualitative features as the baseline case but where the principal can choose y^* as desired.

²⁷The optimal choice of y^* is then the usual monopoly tradeoff: higher y^* may relax the incentives for the group that receives the gift, but fewer agents are offered it and hence exert effort. However, R is non-monotonic in y^* . In words, offering a small gift relaxes the incentive constraint but a large gift renders incentives more difficult and, as $R(1) = 0$, a large enough gift makes incentives impossible. A large gift makes incentives hard because the agent is very unlikely to be assigned to a contractible task. To see this, consider the limiting case where the principal offers a fixed fee of $g = 1 - \beta$. The agent then knows that upon being offered $g = 1 - \beta$ that either $\max\{d_i\} = 1$ or $\max\{b_i\} \geq 1 - \beta$. Bayes Rule implies that the likelihood that the agent is type $\max\{d_i\} = 1$ is close to zero here, and so exerts no effort.

6 Conclusion

Providing incentives and exercising authority are two of the most important roles played by managers. Much of the existing literature is concerned with how an absence of congruence causes authority to harm incentives.²⁸ This paper argues that these congruence effects can be outweighed by better assignment opportunities, such that the interaction depends on the ease of incentive provision - when incentives are easy to provide, authority facilitates incentives but not otherwise. Based on this, it is argued that the empirical evidence is consistent with authority enhancing worker incentives, as the better assignment of workers to tasks may outweigh the distorted assignment incentives of their superiors. Despite this, the paper also suggests a limit to the exercise of authority.

²⁸Also worth noting is Van Den Steen, 2010, where the agent distrusts his boss for a slightly different reason, namely, she holds a different prior to the principal on what should be done. As a result, she believes that the principal makes worse decisions than she would, increasing the cost of incentive provision with authority.

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7 Appendix

Proof of Lemma 1 Consider the task assignment decision of the principal. The principal will either assign the agent to the highest d_i or the highest b_i , so what matters for incentives is the distribution of the first order statistics of the two types of tasks. The cdf of the first order statistic of a unit uniform with n draws is $F(y) = y^n$ with density $f(y) = ny^{n-1}$. The principal's objective at the point of assigning a task is to maximize

$$\max\{\max\{b_i\}, \max\{d_i(1 - \beta)\}\}Ee \quad (16)$$

where Ee is the principal's expectation of the agent's effort. The agent will be assigned to a contractible task if $(1 - \beta)$ of its productivity exceeds the productivity of the best non-contractible task, so that if the agent draws a value z for the best contractible task, the probability that it will be assigned to the agent is given by $F((1 - \beta)z) = ((1 - \beta)z)^n$. But as the agent does not know z when choosing effort, Bayes Law implies that she does to so maximize $e[\beta \int_0^1 z f(z) F((1 - \beta)z) dz - \gamma] = e[\beta \int_0^1 znz^{n-1}((1 - \beta)z)^n dz - \gamma]$ which is

$$e[(1 - \sigma)\beta(1 - \beta)^n \frac{n}{2n + 1} - \gamma]. \quad (17)$$

Incentives can then only be provided if there exists some β such that

$$\beta(1 - \beta)^n \frac{n}{2n + 1} \geq \frac{\gamma}{1 - \sigma}. \quad (18)$$

Computation of Surplus If the agent exerts effort of $e = 1$, there are three possible outcomes.

- The maximum b_i exceeds $1 - \beta$. The (unconditional) surplus created then given by

$$\int_{1-\beta}^1 yny^{n-1} dy = \frac{n}{n + 1}(1 - (1 - \beta)^{n+1}). \quad (19)$$

- The maximum b_i is below $1 - \beta$ and the maximum b_i exceeds the maximum d_i . The (unconditional) surplus created then given by

$$\int_0^{1-\beta} yny^{n-1} \left(\frac{y}{1 - \beta}\right)^n dy = \frac{n}{2n + 1}(1 - \beta)^{n+1}. \quad (20)$$

- The maximum b_i is below $1 - \beta$ and the maximum d_i exceeds the maximum b_i . The (unconditional) surplus created then given by

$$\int_0^1 yny^{n-1} (y(1 - \beta))^n dy = \frac{n}{2n + 1}(1 - \beta)^n. \quad (21)$$

Surplus is then the sum of these three terms.

Proof of Proposition 1 $\frac{d\beta^{**}}{dn} = \frac{-[\log(1-\beta^{**}) + \frac{1}{n(2n+1)}]}{\frac{1}{\beta^{**}} - \frac{n}{1-\beta^{**}}}$. The denominator is positive by the second order condition as we are considering the case where $\beta^{**} < \frac{1}{n+1}$ (see (7)), and reflects simply that the incentives of the agent must be weakly increasing when β^{**} rises.

Proof of Proposition 2 The expected return to exerting effort is given by (17). But as the maximized value of $\beta(1-\beta)^n$ arises at $\beta = \frac{1}{n+1}$, simple substitution yields Proposition 2. Let γ^{**} be the feasible effort cost, namely the effort cost below which the firm can induce effort given the constraint that (7) must hold so $\frac{1}{2n+1}(\frac{n}{n+1})^{n+1} = \frac{\gamma^{**}}{1-\sigma} = \tilde{\gamma}^{**}$. Note that by contrast to the first best, $\frac{d\tilde{\gamma}^{**}}{dn} < 0$, as $\frac{d\tilde{\gamma}^{**}}{dn} = \frac{n}{(2n+1)(n+1)^n}[-\frac{2n}{(2n+1)(n+1)} + \frac{1}{(n+1)^2} + \frac{n}{n+1}\log(\frac{n}{n+1})] < 0$ because $\frac{n}{n+1} < 1$ and $\frac{2n}{2n+1} > \frac{1}{n+1}$. Furthermore, $\gamma^{**} \rightarrow 0$ as $n \rightarrow \infty$.

Proof of Proposition 3 First consider surplus. There are three cases to consider - (i) where the maximum b_i exceeds $1 - \beta$ and hence there is no value of d_i that can beat it, (ii) where where the maximum b_i is below $1 - \beta$ and wins, and (iii) where where the maximum b_i is below $1 - \beta$ and loses to the maximum d_i . Surplus is given by the sum of these three states and is given by

$$S = \int_{1-\beta}^1 nx^{n-1}xdx + \int_0^{1-\beta} xnx^{n-1}(\frac{x}{1-\beta})^n dx + \int_0^1 xnx^{n-1}(x(1-\beta))^n dx. \quad (22)$$

Integration yields

$$S(\beta^{**}) = \frac{n}{2n+1}(1-\beta^{**})^{n+1} + \frac{n}{2n+1}(1-\beta^{**})^n + \frac{n}{n+1}[1 - (1-\beta^{**})^{n+1}] - \gamma. \quad (23)$$

The effect of n on surplus is given by

$$\frac{dS}{dn} = \frac{n}{(n+1)(2n+1)} \frac{d[(1-\beta)^{n+1}]}{dn} - \frac{d[\beta(1-\beta)^n \frac{n}{2n+1}]}{dn} + \frac{2(1-\beta)^{n+1}}{(2n+1)^2} + \frac{1 - (1-\beta)^{n+1}}{(n+1)^2} \quad (24)$$

where $\frac{d[(1-\beta)^{n+1}]}{dn} = (1-\beta)^{n+1}\log(1-\beta) - [(n+1)(1-\beta)^n]\frac{d\beta}{dn}$, $\frac{d[\beta(1-\beta)^n \frac{n}{2n+1}]}{dn} = 0$ from (5), and $\frac{d\beta}{dn} = \frac{-[\log(1-\beta^{**}) + \frac{1}{n(2n+1)}]}{\frac{1}{\beta^{**}} - \frac{n}{1-\beta^{**}}}$. At β close to 0,

$$\frac{dS}{dn} = \frac{2}{(2n+1)^2} > 0 \quad (25)$$

and surplus is enhanced by the principal being better informed. By contrast, remember that the maximum value of β is $\frac{1}{n+1}$. Evaluating (24) at this point, the sign of the effect on total surplus is the sign of $-[\log(1-\beta^{**}) + \frac{1}{n(2n+1)}]$ at $\beta^{**} = \frac{1}{n+1}$, which is always negative.

An Approximate First Best Mechanism in Section 4: Here I show that with sufficient flexibility in the contracting environment, the first best can be achieved. The mechanism consists of the principal making a report of \hat{b} , the highest productivity non-contractible task, and $\{\hat{d}_1, \hat{d}_2, \dots, \hat{d}_n\}$, the entire vector of contractible outcomes, to a mechanism designer, where they are ordered such that \hat{d}_1 is the lowest and \hat{d}_n is the highest. The mechanism is as follows:

- If $\hat{d}_n < \hat{b}$, then
 - With probability ϵ , where ϵ is small, the principal randomly implements one of the 1 to n contractible projects. Let y_i be the observed output if the i th highest element of the \hat{d} vector is implemented. Then if $y_i = \hat{d}_i$, the agent is paid 0, but if $y_i \neq \hat{d}_i$ but $y_i > 0$, then the principal makes a transfer of $T > 0$ to the mechanism designer, where T is large.
 - With probability $1 - \epsilon$, \hat{b} is implemented and the agent is paid a fixed payment of $\beta^{**}\hat{d}_n$, where $\beta^{**}\frac{n}{2n+1}(1 - \epsilon) = \frac{\gamma}{1-\sigma}$.
- If $\hat{d}_n \geq \hat{b}$, implement \hat{d}_n with probability 1 and the agent is offered β^{**} .

Why does this mechanism induce the first best? Begin by assuming that the principal truthfully reveals the realizations of all the contractible variables. Then in the mechanism the principal will choose a non-contractible task if $(1 - \beta^{**})\max\{d_i\} \geq \max\{b_i\} - \beta^{**}\hat{d}_n = \max\{b_i\} - \beta^{**}\max\{d_i\}$ or $\max\{d_i\} \geq \max\{b_i\}$, which yields the first best outcome. Hence, if the principal can be induced to tell the truth over the maximum d , the mechanism designer can impose a penalty (a fixed fee to the agent so has no effect on incentives) for choosing a non-contractible task such that efficient choices are made.

But the principal can be induced to tell the truth about the d vector by random monitoring, where large penalties are imposed if the outcome does not accord with the reported outcome. Then, as in the usual Becker logic, by increasing T and reducing ϵ such that the principal is indifferent about lying about the states, the principal can be induced to tell the truth with (almost) no distortion in task assignments. For any finite return to deviating from d_n to \hat{d}_n , T can be chosen large enough for any ϵ to deter deviation. T , of course needs to be large and so deep pockets are necessary. Finally, note that the agent has incentives to exert effort with incentive payments given by β^{**} . Hence the first best is attainable with these more complex mechanisms and deep pockets for the principal.

Proof of Proposition 4: Consider a more general contract where the agent receives a transfer $\beta(y)$ when output of y is realized. Then as the density of the first order statistic is given by $f(y) = ny^{n-1}$, and the probability of contractible output y “winning” is $(y - \beta(y))^n$, the relevant incentive compatibility constraint is given by

$$\int_0^1 ny^{n-1}\beta(y)(y - \beta(y))^n dy \geq \frac{\gamma}{1 - \sigma} \quad (26)$$

Then consider the principal’s objective. When the principal increases $\beta(y)$ for realized output y , the marginal loss to him is as follows - rather than create surplus of y , instead $y - \beta(y)$ is produced, which occurs whenever *both* the best contractible outcome is y and the best non-contractible outcome is $y - \beta(y)$. Hence the marginal loss is given by $L = -\beta(y)ny^{n-1}n(y - \beta(y))^{n-1}$. Then if λ is the Lagrange multiplier on the incentive constraint, the optimal choice, $\beta^*(y)$ is given by $L = -\lambda \frac{d[\int_0^1 ny^{n-1}\beta(y)(y - \beta(y))^n dy - \frac{\gamma}{1 - \sigma}]}{d\beta(y)}$ or after a small amount of manipulation,

$$\beta^*(y) = \frac{\lambda y}{\lambda(n + 1) + n}. \quad (27)$$

Hence, the unique optimal contract is linear in output, with slope $\frac{\lambda}{\lambda(n+1)+n}$ and so there is no loss from the assumption of linearity above. This formulation also makes intuitive sense where when the incentive constraint is weak, and $\lambda \rightarrow 0$, then $\beta^* \rightarrow 0$, while when the incentive constraint becomes very binding, where $\lambda \rightarrow \infty$, maximum incentives converge to $\beta^* \rightarrow \frac{1}{n+1}$, as above in (7).

More generally, for any common distribution of first order statistics F for both contractible and non-contractible returns, the optimal choice of $\beta^*(y)$ is given by

$$f(y)f(y - \beta^*(y))\beta^*(y) = -\lambda f(y)[F(y - \beta^*(y)) - \beta^*(y)f(y - \beta^*(y))] \quad (28)$$

or

$$\beta^*(y) = -\lambda \left[\frac{F(y - \beta^*(y))}{f(y - \beta^*(y))} - \beta^*(y) \right]. \quad (29)$$

For the distribution of the first order statistic for the uniform with n draws, $\frac{F(y - \beta^*(y))}{f(y - \beta^*(y))} = \frac{y - \beta^*(y)}{n}$ is linear in $\beta(y)$ and so the optimal contract is linear.

Proof of Proposition 5 If a measure m of tasks are done, then if all contractible tasks above d^* are assigned and all non-contractible tasks above b^* , then as $(1 - d^*) + (1 - b^*) = m$, yet $b^* = (1 - \beta)d^*$, so

$$d^* = \frac{2 - m}{2 - \beta}. \quad (30)$$

If the agent knows that the probability she is assigned a task for which she is paid is only $1 - \frac{2-m}{2-\beta}$, the agent's incentive compatibility constraint is given by

$$\beta \frac{2-m}{2(2-\beta)} \geq \frac{\gamma}{1-\sigma}. \quad (31)$$

In order to identify how increasing scale affects the ability to induce effort exertion, consider the case the value of m for which incentives are identical to the case of $n = 1$. This is given by

$$\frac{1-\beta_1}{3} = \frac{1}{2} \left(1 - \left(\frac{2-m^*}{2-\beta_1}\right)^2\right). \quad (32)$$

Note that $m^* < 1$. But $t = \frac{m}{2}$, so the relevant condition in terms of fraction of tasks done is $\frac{1-\beta_1}{3} = \frac{1}{2} \left(1 - \left(\frac{2(1-t^*)}{2-\beta_1}\right)^2\right)$.

When Non-Contractible Tasks Have Higher Expected Return First consider the case where the expected returns from the non-contractible activity are higher, where $B > 1$ while D remains equal to 1. Here the incentive compatibility constraint becomes $\left(\frac{D}{B}\right)^n \beta^{**} (1 - \beta^{**})^n \frac{n}{2n+1} \geq \frac{\gamma}{1-\sigma}$. The maximized value of $\beta^{**} (1 - \beta^{**})^n$ remains $\beta^{**} = \frac{1}{1+n}$, and hence incentives can only be provided if and only if

$$\left(\frac{D}{B}\right)^n \frac{1}{2n+1} \left(\frac{n}{n+1}\right)^{n+1} \geq \frac{\gamma}{1-\sigma}. \quad (33)$$

This is harder to satisfy than before as $D < B$. Hence the problems that plague effort exertion in the baseline hold with greater force in this situation.

Proof of Proposition 6: Initially consider the outcome as $n \rightarrow \infty$. In the limit, the principal receives a first order statistic of D from the contractible distribution and B from the non-contractible one. The first best arises here if $D(1-\beta) \geq B$, as the agent is always assigned a contractible task. If the agent believes that she is always assigned the contractible task, then the lowest possible value that β can take to satisfy incentive compatibility is given by $\beta = \frac{\gamma}{D(1-\sigma)}$. Then the equilibrium inference is indeed true if the agent is always allocated to the contractible task, which occurs if and only if $D - \frac{\gamma}{1-\sigma} \geq B$.

Why begin with the first best condition? In the first best outlined above, the share that the agent receives is at its lowest possible level because she is always rewarded if she exerts effort. If $D - \frac{\gamma}{1-\sigma} < B$, the principal always distorts job assignments and β must rise to compensate as the agent is not always rewarded for exerting effort. But this guarantees that $D(1-\beta) < B$ for any smaller n if $D - \frac{\gamma}{1-\sigma} < B$, in which case increases in n tighten the feasibility constraint. The fact that D exceeds B may relax the incentive constraint, but the

logic of incentives ultimately failing in n continues to hold. As a result, the results of the previous section generalize to these settings.

Proof of Lemma 3 The conditional probability that $\max\{d_i\} \geq y^*$ given the fixed fee being offered is given by

$$\frac{1 - y^{*n}}{[1 - y^{*n}] + y^{*n}[1 - ((1 - \beta)y^*)^n]} = \frac{1 - y^{*n}}{1 - (1 - \beta)^n y^{*2n}}, \quad (34)$$

and the incentive compatibility constraint is then given by

$$\beta \left(\frac{1 - y^{*n}}{1 - (1 - \beta)^n y^{*2n}} \right) \int_{y^*}^1 z \frac{nz^{n-1}}{1 - y^{*n}} ((1 - \beta)z)^n dz \geq \gamma, \quad (35)$$

or

$$\frac{1 - y^{*(2n+1)}}{1 - (1 - \beta)^n y^{*2n}} \beta (1 - \beta)^n \frac{n}{2n + 1} \geq \gamma. \quad (36)$$

Proof of Proposition 7:

Freschet The Freschet has cdf given by $F(x) = \exp\{-(\frac{x}{s})^{-\alpha}\}$, $\alpha > 1$ and $x \geq 0$. The mean of this distribution is given by $s\Gamma(1 - \frac{1}{\alpha})$, where Γ is the gamma function. Now consider the first order statistic of this Freschet with n draws. It is Freschet with the only change from the initial distribution being that s becomes $s_n = s(n^{\frac{1}{\alpha}})$. For simplicity consider the case where $s = 1$. The distribution of the first order statistic is then Freschet with mean $n^{\frac{1}{\alpha}}\Gamma(1 - \frac{1}{\alpha})$ and variance that is $(n^{\frac{1}{\alpha}})^2$ times the variance of the base distribution. The distribution of the first order statistic is distributed identically to $n^{\frac{1}{\alpha}}$ times the distribution of the initial distribution. What this implies is that the likelihood that the non-contractible task beats $(1 - \beta)$ times the contractible task is independent of n because if $\max\{d_i\}(1 - \beta) \geq \max\{b_i\}$, then $n^{\frac{1}{\alpha}}\max\{d_i\}(1 - \beta) \geq n^{\frac{1}{\alpha}}\max\{b_i\}$ and so n plays no role.

Gumbel The Gumbel distribution is given by $F(x) = \exp^{-\exp(x)}$ where $-\infty \leq x < \infty$. This distribution has mean η , where $\eta = 0.5772$. The first order statistic from n draws of this distribution is Gumbel with $F(x) = \exp^{-\exp(x - \log(n))}$ with mean $0.5772 + \log(n)$, but with unchanged other moments. As a result, the only change from adding more observations is to increase the mean - the shape of the distribution remains unchanged. As a result, the probability of being assigned to a contractible tasks for a given β is independent of n